Subject Name: SIGNALS AND SYSTEMS

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Year and Sem, Department: II/I ECE-A

Unit-I: SIGNAL ANALYSIS

IMPORTANT POINTS:

1. A **signal** is defined as a time varying signal which contains some information. A Signal is a function of one or more independent variables. Signals may be of continuous time or discrete time signals.

2. Signals on telephone wires, video signal, voice signal, EEG, ECG etc are some of the examples of the signals.

3. The following are the Basic Signals:

- Unit Step Signal
- Unit Impulse Function
- Ramp Signal
- Parabolic Signal
- Signum Function
- Exponential Function
- Rectangular Signal
- Triangular Signal
- Sinusoidal Signal

4. Signals are classified into the following categories:

- Continuous Time and Discrete Time Signals
- Deterministic and Non-deterministic Signals
- Even and Odd Signals
- Periodic and Aperiodic Signals
- Energy and Power Signals
- Real and Imaginary Signals

5. The following operation can be performed with amplitude:

- Amplitude Scaling
- Addition

- Subtraction
- Multiplication

6. The following operations can be performed with time:
   - Time shifting
   - Time Scaling
   - Time Reversal

7. A **System** is defined as an entity that acts on an input signal and transforms it in to an output signal. For one or more inputs, the system can have one or more outputs.
Example: Communication System

8. Systems are classified into the following categories:

- Liner and Non-liner Systems
- Time Variant and Time Invariant Systems
- Liner Time variant and Liner Time invariant systems
- Static and Dynamic Systems
- Causal and Non-causal Systems
- Invertible and Non-Invertible Systems
- Stable and Unstable Systems

There is a perfect analogy between vectors and signals.

9.**Vector**
A vector contains magnitude and direction. The name of the vector is denoted by bold face type and their magnitude is denoted by light face type.
The vector V_1 can be expressed in terms of vector V_2

$$V_1 = C_{12}V_2 + Ve$$

Where V_e is the error vector.
But this is not the only way of expressing vector V_1 in terms of V_2. The alternate possibilities are:

$$V_1 = C_1V_2 + V_{e1}$$
$$V_2 = C_1V_2 + V_{e2}$$

10) The error signal is minimum for large component value. If C12=0, then two signals are said to be orthogonal.

Dot Product of Two Vectors

$$V_1 \,.\, V_2 = V_1.V_2 \cos\theta$$

θ = Angle between V_1 and V_2

$$V_1.\, V_2 = V_2.V_1$$

11**. Signal**

The concept of orthogonality can be applied to signals. Let us consider two signals $f_1(t)$ and f2(t). Similar to vectors, you can approximate $f_1(t)$ in terms of $f_2(t)$ as

$f_1(t) = C_{12}\, f_2(t) + f_e(t)$ for ($t_1 < t < t_2$)

$\Rightarrow f_e(t) = f_1(t) - C_{12}\, f_2(t)$

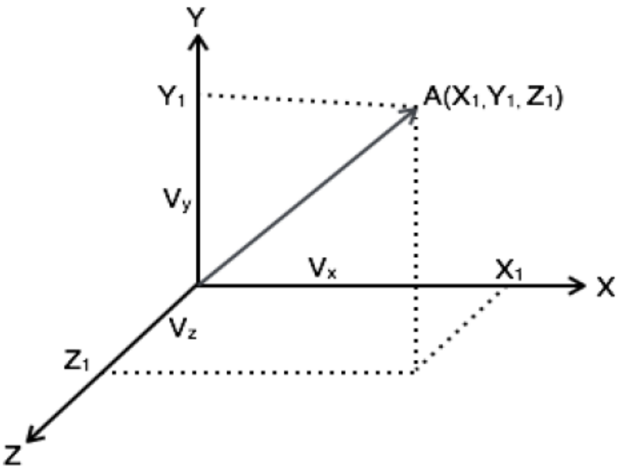
One possible way of minimizing the error is integrating over the interval t_1 to t_2.

$$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} [f_e(t)]dt$$

$$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} [f_1(t) - C_{12}f_2(t)]dt$$

12 .Orthogonal Vector Space :

A complete set of orthogonal vectors is referred to as orthogonal vector space. Consider a three dimensional vector space as shown below:



Consider a vector A at a point (X_1, Y_1, Z_1). Consider three unit vectors (V_X, V_Y, V_Z) in the direction of X, Y, Z axis respectively. Since these unit vectors are mutually orthogonal, it satisfies that

$$V_X \,.\, V_X = V_Y \,.\, V_Y = V_Z \,.\, V_Z = 1$$

$$V_X \,.\, V_Y = V_Y \,.\, V_Z = V_Z \,.\, V_X = 0$$

13. Orthogonal Signal Space :

Let us consider a set of n mutually orthogonal functions $x_1(t)$, $x_2(t)$... $x_n(t)$ over the interval t_1 to t_2. As these functions are orthogonal to each other, any two signals $x_j(t)$, $x_k(t)$ have to satisfy the orthogonality condition. i.e.

$$\int_{t_1}^{t_2} x_j(t)x_k(t)dt = 0 \ \ \text{where}\, j \neq k$$

$$\text{Let} \int_{t_1}^{t_2} x_k^2(t)dt = k_k$$

14. The average of square of error function $f_e(t)$ is called as mean square error. It is denoted by ε (epsilon).

$$\varepsilon = \frac{1}{t_2 - t_1}[\int_{t_1}^{t_2}[f^2(t)]dt + (C_1^2K_1 + C_2^2K_2 + \ldots + C_n^2K_n)]$$

15.Orthogonality in Complex Functions:

If $f_1(t)$ and $f_2(t)$ are two complex functions, then $f_1(t)$ can be expressed in terms of $f_2(t)$ as $f1(t)=C12f2(t)$.. with negligible error

$$\text{Where}\ C_{12} = \frac{\int_{t_1}^{t_2} f_1(t)f_2^*(t)dt}{\int_{t_1}^{t_2} |f_2(t)|^2 dt}$$

Where $f_2^*(t)$ is the complex conjugate of $f_2(t)$

If $f_1(t)$ and $f_2(t)$ are orthogonal then $C_{12} = 0$

2-Marks Questions:

1) Define even and odd components of the signal how do you get it. 2M (NOV-2016)
2) Sketch the unit step function and signum function bring the relation between them. 3M (NOV-2016)
3) Write the Fundamental difference between Continuous and Discrete time signals. 2M (NOV-2016)
4) How do you approximate a signal using orthogonal functions? 3M (Nov-2017_R-15)
5) Write about unit step function and unit impulse function. 2M (Nov-2017_R-16)
6) Write the differences between the continuous-time signal $e^{j\omega 0t}$ and the discrete-time signal $e^{j\omega 0n}$ 2M (Nov-2017_R-16)
7) Define Mean Square error? 2M (Mar-2017-R-13)
8) What is an Orthognal Function? 3M (Mar-2017-R-13)
9) What is a Signum Function? 2M (Mar-2017-R-13)
10) Determine whether a unit step signal u (t) is energy or power signal. 2M (Mar-2017-R-15)
11) Define principle of orthogonality. 3M (Mar-2017-R-15)
12) Define Unit step function and Signum function 3M(April-2018-R-16)

13) What is meant by Total response?2M (April-2018-R-16)

5-Marks Questions:

1) Explain orthogonality property between two complex functions f1(t) and f2(t) for a real variable t. (Nov-2016)
2) Define and derive the expression for evaluating mean square errors and its types. (Nov-2016) (Mar-2017-R-13)
3) Derive from the basics, how any continuous time signal x(t) can be represented as an integral of impulses. 5M (Nov-2017_R-15).
4) Discuss the orthogonality in complex signals. 5M (Nov-2017_R-15)
5) Define orthogonal signal space and orthogonal vector space. Bring out clearly its applications in representing a signal and vector respectively. 5M (Nov-2017_R-16)
6) Explain how functions can be approximated using orthogonal functions. 5M (Nov-2017_R-16)
7) Explain about complete set of Orthognal functions? (Mar-2017-R-13)
8) Test the orthogonality of the signals sin wt cos2 wt over the interval (t_0 to t_0+T). (Mar-2017-R-15)
9) Define the error function while approximating signals and hence derive the expression for condition for orthogonality between two waveforms f_1(t) and f_2(t) (April-2018-R-16)

CHOOSE THE CORRECT ANSWER

UNIT – I: SIGNAL ANALYSIS

1. Graphical representation of a signal in time domain is called ( )
   (a) Frequency spectrum (b) frequency (c) waveform (d) none of the above
2. Graphical representation of a signal in frequency domain is called ( )
   (a) Frequency spectrum (b) frequency (c) waveform (d) none of the above
3. Analog- signals can be converted into discrete-time signals by ( )
   (a) Sampling (b) coding (c) quantizing (d) none of the above
4. u(t - a) = 0, if ( )
   a. (a) t - a = 0 (b) t - a < 0 (c) t - a > 0 (d)) t > a
5. tδ(t)= ( )
   b. (a) t (b) 0 (c) 1 (d) u(t)
6. A deterministic signal has ( )
   (a) no uncertainty (b) uncertainty (c) partial uncertainty (d) none of the above
7. The relation between a signum function and a unit step function is, sgn (t) = ( )
   (a) 2u (t) - 1 (b) u (t) - 1 (c) 2u (t) (d) u(t) – u(-t)
8. x (t) = u(t) - u(t - 10) is a ( )
   (a) Power signal (b) energy signal (c) neither (a) or (b) (d) both (a) and (b)
9. x(t) = e^{-5t} u(t) is a ( )
   (a) Power signal (b) energy signal (c) neither (a) or (b) (d) both (a) and (b)
10. A signal is an energy signal if ( )
   (a) E = 0, P = 0 (b) E = 0, P = finite (e) E = finite, P = 0 (d) E = finite, P = 0

Answers

1	C	3	A	5	B	7	A	9	B
2	A	4	B	6	A	8	B	10	C

UNIT-II
FOURIER SERIES AND FOURIER TRANSFORMS

1. Fourier series:

To represent any periodic signal x(t), Fourier developed an expression called Fourier series. This is in terms of an infinite sum of sines and cosines or exponentials. Fourier series uses orthoganality condition.

2. Fourier Series Representation of Continuous Time Periodic Signals :

A signal is said to be periodic if it satisfies the condition x (t) = x (t + T) or x (n) = x (n + N).

Where T = fundamental time period,
ω_0= fundamental frequency = 2π/T

3. There are two basic periodic signals:
$x(t)=\cos\omega 0t$(sinusoidal) &
$x(t)=ej\omega 0t$(complex exponential)

These two signals are periodic with period $T=2\pi/\omega 0$

3.The Coefficients of Fourier series are a_0, a_n, b_n

4. Properties of Fourier series are as follows:

- Linearity property
- Time Shifting Property
- Frequency Shifting Property
- Time reversal Property
- Time Scaling Property
- Differentiation and Integration Properties
- Multiplication and Convolution Properties
- Conjugate and Conjugate Symmetry Properties

5.There are different types of Fourier Series:

- Trigonometric Fourier Series

$$\therefore x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \quad (t_0 < t < t_0 + T)$$

- Exponential Fourier Series

$$\therefore f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \qquad (t_0 < t < t_0 + T).$$

6. The following are the Dirichlet conditions:

Dirichlet's Conditions

The conditions, under which a periodic function $f(t)$ can be expanded in a convergent Fourier series, are known as Dirichlet's conditions.

These are as follows:

i. $f(t)$ is a single valued function.

ii. $f(t)$ has a finite number of discontinuities in each period, T.

iii. $f(t)$ has a finite number of maxima and minima in each period, T.

iv. The integral, $\int_0^T |f(t)|\,dt$ exists and is finite or in other way,

$\int_0^T [f(t)]^2\,dt < \infty$.

7. Fourier Transform:

The main drawback of Fourier series is, it is only applicable to periodic signals. There are some naturally produced signals such as nonperiodic or aperiodic, which we cannot represent using Fourier series. To overcome this shortcoming, Fourier developed a mathematical model to transform signals between time (or spatial) domain to frequency domain & vice versa, which is called 'Fourier transform'.

8. Fourier transform has many applications in physics and engineering such as analysis of LTI systems, RADAR, astronomy, signal processing etc.

9. Conditions for Existence of Fourier Transform:

Any function f(t) can be represented by using Fourier transform only when the function satisfies Dirichlet's conditions. i.e.

- The function f(t) has finite number of maxima and minima.
- There must be finite number of discontinuities in the signal f(t),in the given interval of time.
- It must be absolutely integrable in the given interval of time i.e.

$$\int_{-\infty}^{\infty} |f(t)|\,dt < \infty$$

10. . Properties of Fourier Transform are as follows:

- Linearity property

- Time Shifting Property
- Frequency Shifting Property
- Time reversal Property
- Time Scaling Property
- Differentiation and Integration Properties
- Multiplication and Convolution Properties
- Conjugate and Conjugate Symmetry Properties

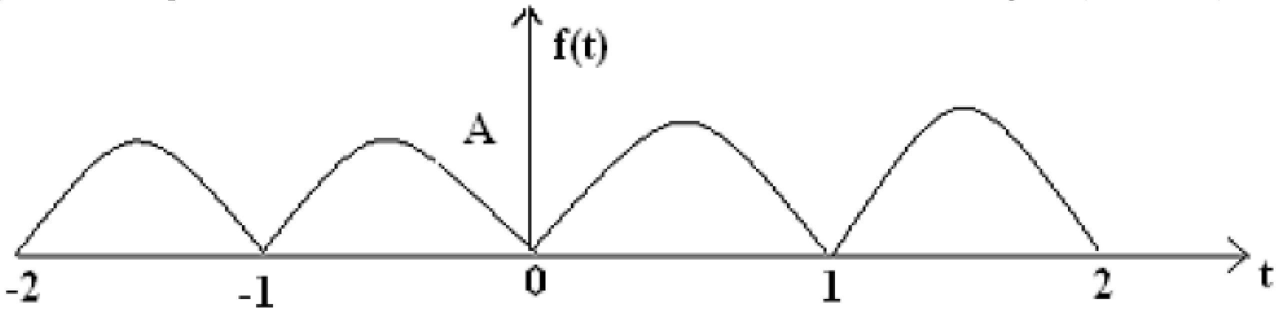
11. The Hilbert transform defined in the time domain is a convolution between the Hilbert transformer 1/ (π t) and a function f(t).

2-Marks Questions:

1) Distinguish between Series and Transform in the Fourier representation of a signal. 2M (NOV-2016)
2) Write the Convolution property of Fourier Transform. 2M (NOV-2016)
3) If the Fourier series coefficient of x(t) is Cn, find the Fourier series coefficient of x*(t) . 2M (Nov-2017_R-15)
4) Determine the Fourier transform of $x(t) = e^{-at} (\cos\Omega_0 t)\, u(t)$ 3M (Nov-2017_R-15)
5) Determine the complex exponential Fourier series representation for $x(t) = \cos(2t + \pi/4)$ 2M (Nov-2017_R-15)
6) Find the Fourier transform of $x(t) = e^{j\omega_0 t}$. 3M (Nov-2017_R-16)
7) Compare Fourier series and Fourier transform. 3M (Mar-2017-R-15)
8) State "time shift" property of Fourier transform? (April-2018-R-16)
9) State any two properties of Fourier series. (May-2019-R-16)
10) Find the Fourier transform of the signal x(t) = 20 sinc (20t). (May-2019-R-16)

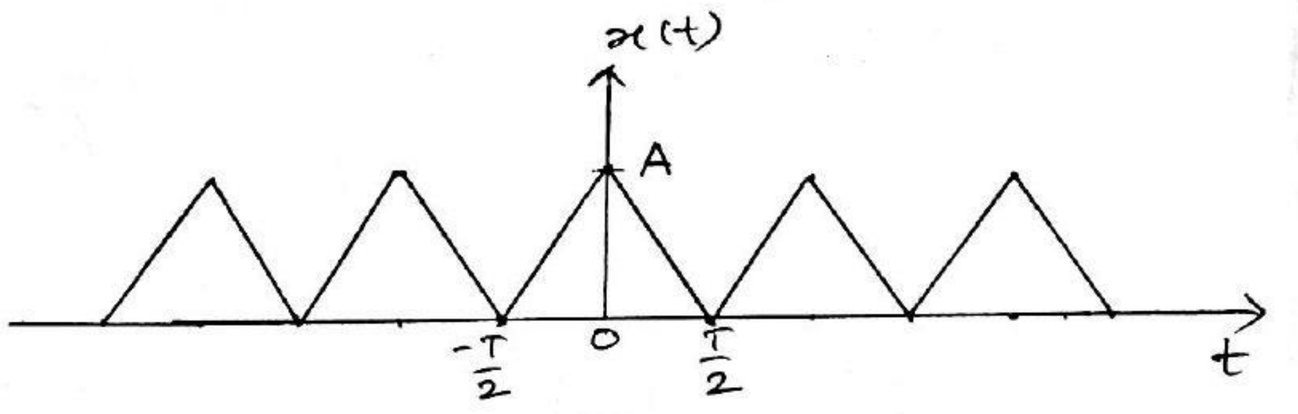
5-Marks Questions:

1) Find the Exponential Fourier series for the rectified Sine wave as shown in figure. (Nov-2016)



2) Obtain the Fourier transform of the following functions: (Nov-2016)
a) Impulse Signal b) Single symmetrical Gate Pulse.

3) Determine the exponential form of the Fourier series representation of the signal shown below. **Nov-2017_R-15**



4) Find the Fourier transform of Double sided exponential Signal x(t)= $e^{-/t/}$ **Nov-2017_R-15**
5) State and Prove the Convolution property of Fourier transform. (Nov-2017_R-16)
6) Expand following function f(t) by trigonometric Fourier series over the interval (0,1). In this interval f(t) is expressed as f(t) = At (Nov-2017_R-16)
7) State and prove multiplication property of continuous time Fourier series. (Nov-2017_R-16)
8) Find the Fourier transform of symmetrical gate pulse and sketch the spectrum. (Nov-2017_R-16)
9) Explain about the Trignometric Fourier Series? (Mar-2017-R-13)
10) Write about the complex fourier spectrum? (Mar-2017-R-13)
11) State and Prove the time shifting and frequency shifting properties of Fourier transform? (Mar-2017-R-13)
12) Derive the expression for trigonometric Fourier series coefficients. (Mar-2017-R-15)
13) State the dirichilet's conditions for existence of Fourier series. (Mar-2017-R-15)
14) Find the exponential Fourier series of the signal x(t) = 5cos 5t+10 sin 15t (Mar-2017-R-15)
15) Find the Fourier transform of x(t) =e^{-at} u (t). (Mar-2017-R-15)
16) State and prove the convolution property of Fourier transform. (Mar-2017-R-15)
17) State and prove parsavel's energy theorem. (Mar-2017-R-15)
18) If x (t) has Fourier transform pair X (w). Deduce the Fourier Transform of X (at-t_0) (Mar-2017-R-15)
19) Obtain the expressions to represent trigonometric Fourier coefficients in terms of exponential Fourier coefficients. (April-2018-R-16)
20) State and prove Differentiation and integration properties of Fourier Transform. (April-2018-R-16)
21)Obtain the Fourier series coefficients for x(t) = A Sin $\omega_0 t$. (May-2019-R-16)
22) Define Fourier transform. Explain the properties of Fourier transform. (May-2019-R-16)

CHOOSE THE CORRECT ANSWER

1. For the existence of Fourier series, Dirichlet's conditions are ( )
(a) necessary (b) sufficient (c) necessary and sufficient (d) none of these
2. In the trigonometric Fourier series representation of a signal, ao (or Ao) is the ( )
(a) RMS value (b) mean square value (c) peak value (d) average value
3. The trigonometric Fourier series representation of a function with half wave consists of( )
(a) Cosine terms only (c) odd harmonics (b) sine terms only (d) even harmonics
4. The most widely used Fourier series is ( )
(a) trigonometric series (c) exponential series (b) cosine series (d) none

5. A trigonometric Fourier series has ( )
(a) a one-sided spectrum (b) a two-sided spectrum (c) both one-sided and two-sided spectrums (d) none
6. An exponential Fourier series has ( )
(a) a one-sided spectrum (b) a two-sided spectrum (c)both one-sided and two-sided spectrums (d) none
7. The frequency spectrum of a periodic signal is ( )
(a) continuous (b) discrete (c) both continuous and discrete (d) none
8. The net areas of sinusoids over complete periods are ( )
(a) finite (b) infinite (c) zero(d) none of these
9. The phase spectrum of exponential Fourier series is about vertical axis. ( )
(a) symmetrical (b) antisymmetrical (c) both (a) and (b) (d) none
10. The phase spectrum of exponential Fourier series is about vertical axis. ( )
(a) symmetrical (b) antisymmetrical (c) both (a) and (b) (d) none

Answers

1	B	3	C	5	A	7	B	9	A
2	D	4	C	6	B	8	C	10	B

UNIT-III
SIGNAL TRANSMISSION THROUGH LINEAR SYSTEMS

1. A system is said to be linear when it satisfies superposition and homogenate principles. Consider two systems with inputs as x1(t), x2(t), and outputs as y1(t), y2(t) respectively. Then, according to the superposition and homogenate principles,

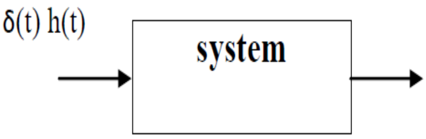
$$T\,[a_1\, x_1(t) + a_2\, x_2(t)] = a_1\, T[x_1(t)] + a_2\, T[x_2(t)]$$

$$\therefore T\,[a_1\, x_1(t) + a_2\, x_2(t)] = a_1\, y_1(t) + a_2\, y_2(t)$$

From the above expression, is clear that response of overall system is equal to response of individual system.

2. Impulse Response:

The impulse response of a system is its response to the input δ(t) when the system is initially at rest. The impulse response is usually denoted h(t). In other words, if the input to an initially at rest system is δ(t) then the output is named h(t).



3.If a system is both liner and time variant, then it is called liner time variant (LTV) system.

4.If a system is both liner and time Invariant then that system is called liner time invariant (LTI) system.

5. The ***impulse response*** h(t) of a continuous-time LTI system (represented by **T)** is defined to be the response of the system when the input is δ(t), that is,

$$\mathbf{h(t) = T\{\, \delta(t)\}}$$

6. Transmission is said to be distortion-less if the input and output have identical wave shapes. i.e., in distortion-less transmission, the input x(t) and output y(t) satisfy the condition:

$$y\,(t) = Kx(t - t_d)$$

Where t_d = delay time and
k = constant.

7. One of the most basic operations in any signal processing system is filtering. Filtering is the process by which the relative amplitudes of the frequency components in a signal are changed or perhaps some frequency components are suppressed

8. An ideal frequency-selective filter is one that exactly passes signals at one set of frequencies and completely rejects the rest. The band of frequencies passed by the filter is referred to as the pass band, and the band of frequencies rejected by the filter is called the stop band.

9. An ideal low-pass filter (LPF) is specified by

$$|H(\omega)| = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

The frequency ***wc*** is called the cutoff frequency.

10. An ideal high-pass filter (HPF) is specified by

$$|H(\omega)| = \begin{cases} 0 & |\omega| < \omega_c \\ 1 & |\omega| > \omega_c \end{cases}$$

11. An ideal bandpass filter (BPF) is specified by

$$|H(\omega)| = \begin{cases} 1 & \omega_1 < |\omega| < \omega_2 \\ 0 & \text{otherwise} \end{cases}$$

12. An ideal bandstop filter (BSF) is specified by

$$|H(\omega)| = \begin{cases} 0 & \omega_1 < |\omega| < \omega_2 \\ 1 & \text{otherwise} \end{cases}$$

2-Marks Questions:

1. Characterize a Linear Time Invariant (LTI) System.(Dec-2016-R-15)
2. Express and derive the Relationship between Bandwidth and Rise time .(Dec-2016-R-15)
3.Define system bandwidth? (Mar-2017 R-13)
4.what is causality? (Mar-2017 R-13)
5. Explain with suitable example what is meant by an LTI system. (Mar-2017 R-15)
6. Define system Bandwidth and signal Bandwidth. (Mar-2017 R-15)
7. Give the relationship between bandwidth and rise time of a signal. (Nov-2017 R-15)
8. The input and impulse response of continuous time systems are given below. Find the output of continuous time systems. $x(t) = e^{-3t}$ u(t), h(t) = u(t-1) (Nov-2017 R-15)
9. Define signal bandwidth and system bandwidth. 5M (Nov-2017-R-16)
10. Give the condition for the physical reliability of a system.(May-2019)

5-Marks Questions:

1. Explain the difference between the following systems with examples. (Dec-2016-R-15)
   a) Linear and Non-linear systems. b) Causal and Non-Causal systems.
2. Define Time invariant and shift invariant systems and given the system function of a LTI system be 1/jw+2 evaluate the output of the system for an input $(0.9)^t$ u (t). (Dec-2016-R-15)
3.(a) Write about the relationship between bandwidth and rise time in linear system?(Mar-2017 R-13)
   (b) Explain about the transfer function of LTI System?
4(a) Explain about the impulse response of Linear system? (Mar-2017 R-13)
   (b) Write about the Parley-Weiner criterion for physical realization of system?

5 (a) Define Transfer function and state its relation with Impulse function. 3M (Mar-2017-R-15)
(b) Find the impulse response of a continuous time LTI system with 7M (Mar-2017-R-15)
H(s) = S-1/(S+1)(S+2) such that i) Re[S] >2 ii) -1 Re[S]<2

6.(a) Derive the relation between Bandwidth and Rise time. 5M (Mar-2017-R-15)
b) Determine whether the system governed by the equation y(n)= 5x(n) is linear or not Assume that x(n) represents the input to the system and y(n) represents its output. 5M (Mar-2017-R-15)
7.Write short notes on the following. 5M (Nov-2017-R-15)
(a) Ideal filters characteristics.
(b) Filter characteristics of a linear system.
8 (a)Find the transfer function of the system governed by the following impulse response.5M(Nov-2017-R-15)

$$h(t) = u(t) + 0.5e^{-6t} u(t) + 0.2e^{-3t} \cos t\, u(t).$$

(b) Check whether the following system is linear, casual and time invariant or not. 5M(Nov-2017-R-15)

$$d^3y(t)/dt^3 + 4d^2y(t)/dt^2 + 5dy(t)/dt + 2y^2(t) = x(t).$$

9. Derive the relationship between rise time and bandwidth. 5M (Nov-2017-R-16)
10. What is an LTI system? Explain the properties of it. 5M (May-2019)

CHOOSE THE CORRECT ANSWER

1. For distortion less transmission, ( )

(a) $y(t) = kx(t)$ (b) $y(t) = x(t - t_d)$ (c) $y(t) = kx(t - t_d)$ (d) none of these

2. For defining a transfer function, the initial conditions must be taken as ( )

(a) zero (b) infinite (c) finite (d) none of these

3. For distortion less transmission, the amplitude response is ( )

(a) zero (b) infinite (c) constant (d) linear

4. For distortion less transmission, the phase response is ( )

(a) constant (b) linear (c) zero (d) nonlinear

5. for distortion less transmission, system bandwidth must be equal to ( )

(a) Signal bandwidth (b) two times signal bandwidth (c) infinite (d) 1/2 of signal bandwidth

6. The convolution of x(t) and h(t) is given by $y(t) = \int_0^t x(\tau)\, h(t-\tau)\, d\tau$ then ( )

(a) both x(t) and h(t) are causal (b) both x(t) and h(t) are non-causal
(c) x(t) is causal and h(t) is non-causal (d) h(t) is causal and x(t) is non-causal

7. The spectral density of white noise is ( )

(a) Exponential (b) Uniform
(c)Poisson (d) Gaussian

Answers

1	C	2	A	3	C	4	B	5	A
6	A	7	B						